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Question 2

Intent of Question

The primary goals of this question were to assess a student's ability to (1) calculate a probability; (2) assess whether a claim about randomness is questionable in light of a calculated probability; and (3) determine whether a description of a simulation method achieves a correct simulation of a random process.

Solution

Part (a):

The probability that all 3 people selected are women can be calculated using the multiplication rule, as follows:

P(all three selected are women)

$= P(\text{first is a woman}) \times P(\text{second is a woman} \mid \text{first is a woman}) \times P(\text{third is a woman} \mid \text{first two are women})$

$= \frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} \approx 0.012$

Part (b):

The probability calculated in part (a) does provide a reason to doubt the manager's claim that the selections were made at random. The calculation shows that there is only about a 1.2% chance that random selection would have resulted in three women being selected. The probability is small enough that it may cast doubt on the manager's claim that the selections were made at random.

Part (c):

No, the process does not correctly simulate the random selection of three women from a group of nine people of whom six are men and three are women. The random selection of three people among nine is done *without* replacement. However, in the simulation with the dice, the three dice rolls in any given trial are independent of one another, indicating a selection process that is done *with* replacement.

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Question 2 (continued)

Scoring

Parts (a), (b), and (c) were scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response correctly computes the probability of selecting the three women, and shows how the probability was computed.

Partially correct (P) if the response shows only one of the following:

Gives the correct probability of $\frac{1}{84}$ (0.012 or 0.011 is acceptable) but does not show how it was computed;
OR
Correctly shows how the probability should be computed, but does not carry the computation through correctly;
OR
Correctly computes (showing work) only the numerator, or only the denominator of the correct answer. (For example, $\frac{1}{9}\times\frac{1}{8}\times\frac{1}{7}\approx 0.002$, or $\frac{3}{9}\times\frac{2}{9}\times\frac{1}{9}\approx 0.008$, or $\frac{3}{9}\times\frac{3}{8}\times\frac{3}{7}\approx 0.054$);
OR
Mistakenly assumes independence and calculates (showing work) the binomial probability $\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}=\frac{1}{27}\approx 0.037$.

Incorrect (I) if the response does not meet the criteria for E or P.

Part (b) is scored as follows:

Essentially correct (E) if the response states that the probability from part (a) is small (or insufficiently small), makes an appropriate decision consistent with the probability being small (or insufficiently small), and does so in the context of this situation.

Partially correct (P) if the response shows only one of the following:

Otherwise satisfies the criteria for an E but does so without any context;
OR
States a significance level and makes a decision in context that is appropriate to the given probability in part (a) and the stated significance level, but does not explicitly compare the probability and the significance level;
OR
Otherwise satisfies the criteria for an E but does not explicitly make a decision about whether there is reason to doubt the manager's claim. (For example: "The probability of selecting the three women from among the nine employees is very small so it is unlikely to occur by chance.")

Incorrect (I) if the response does not meet the criteria for E or P.

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Question 2 (continued)

Notes:

- Each of the following situations is one in which a response that otherwise would be scored as E should be scored as P, and a response that otherwise would be scored as P should be scored as I:
  - The response includes a statement that the small probability *proves* that the manager did not make the selection at random (or any equivalent wording).
  - The response includes a statement that clearly interprets the probability from part (a) to be the probability that the manager selected the three people at random.
- Each of the following situations is one in which the response is scored as I:
  - The decision is inconsistent with the justification (*e.g.*, "The probability is very small, so there is no reason to doubt the manager's claim").
  - The response states or implies that because the selection of three women was *not impossible,* there is *no reason to doubt* the manager's claim.

Part (c) is scored as follows:

Essentially correct (E) if the response answers no *AND* states that the dice outcomes in the proposed simulation are independent *AND* states that the genders of the selected convention attendees are dependent. The table below shows statements that should be considered equivalent to the required statements of independence and dependence.

Independence of dice outcomes	Dependence of genders
• The three dice outcomes are independent. • The probability of rolling a 5 or a 6 is the same on all three dice. • The dice simulation actually simulates sampling with replacement.	• The genders of the three people are dependent (or not independent). • The probability of selecting a woman changes after each selection. • The people are sampled without replacement.

OR

Essentially correct if the response answers no *AND* computes the correct probability that a trial of the simulation will indicate the selection of three women $\left(\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\approx 0.037\right)$ *AND* states that the probability is different from the probability found in part (a).

Partially correct (P) if the response correctly answers no and either:

States only that the dice outcomes are independent or states only that the genders of the selected convention attendees are dependent, but not both;

OR

Otherwise meets the criteria for E but has poor communication. An example of poor communication is: "No, because it selects with replacement. It isn't possible for the same person to be selected twice." (There is an apparent shift between the two sentences from describing the simulation to describing the actual selection of people, but that is not made clear.)

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Question 2 (continued)

Incorrect (I) if the response does not meet the criteria for E or P.

Note: Pointing out that a sample of three people is more than 10% of the population of nine people should be considered equivalent to stating that the selection of a woman is not independent among the three people selected to attend the convention.

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and one part incorrect

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct and two parts incorrect

OR

Two parts partially correct and one part incorrect

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2. Nine sales representatives, 6 men and 3 women, at a small company wanted to attend a national convention. There were only enough travel funds to send 3 people. The manager selected 3 people to attend and stated that the people were selected at random. The 3 people selected were women. There were concerns that no men were selected to attend the convention.

(a) Calculate the probability that randomly selecting 3 people from a group of 6 men and 3 women will result in selecting 3 women.

$(3/9)(2/8)(1/7) = .012$

(b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

There is a small chance that these results would occur if the selection was random, therefore there is reason to doubt the manager's claim.

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2 A2

(c) An alternative to calculating the exact probability is to conduct a simulation to estimate the probability. A proposed simulation process is described below.

> Each trial in the simulation consists of rolling three fair, six-sided dice, one die for each of the convention attendees. For each die, rolling a 1, 2, 3, or 4 represents selecting a man; rolling a 5 or 6 represents selecting a woman. After 1,000 trials, the number of times the dice indicate selecting 3 women is recorded.

Does the proposed process correctly simulate the random selection of 3 women from a group of 9 people consisting of 6 men and 3 women? Explain why or why not.

The proposed process does not simulate the selection correctly because it assumes the probability ~~that~~ that a woman will be selected is 1/3 and that the trials are independent of each other, both of which are not true.

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2 B1

2. Nine sales representatives, 6 men and 3 women, at a small company wanted to attend a national convention. There were only enough travel funds to send 3 people. The manager selected 3 people to attend and stated that the people were selected at random. The 3 people selected were women. There were concerns that no men were selected to attend the convention.

(a) Calculate the probability that randomly selecting 3 people from a group of 6 men and 3 women will result in selecting 3 women.

$P = {}^3/_9 \left({}^2/_9\right)\left({}^1/_9\right) = .00823$

(b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

Yes, there is reason to doubt the manager's claim because w/ a probability of .00823, it is highly unlikely that out of a pick of 3, all 3 women were chosen. Even though it is possible, one would expect doubt to arise w/ a .00823 chance of all 3 women being picked randomly.

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Does the proposed process correctly simulate the random selection of 3 women from a group of 9 people consisting of 6 men and 3 women? Explain why or why not.

No, the proposed simulation does not simulate the random selection because each time you roll a die you have a 2/6 or 1/3 chance of rolling a 5 or a 6, which the proposed process says represents women. However, in the actual random selection of men & women the probability of choosing a woman goes down after one has already been selected because the # of women to choose from has now decreased. So instead of a 3/9 chance you go down to a 2/9 chance once one woman has been selected, diminishing the odds of picking another woman.

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2 C1

2. Nine sales representatives, 6 men and 3 women, at a small company wanted to attend a national convention. There were only enough travel funds to send 3 people. The manager selected 3 people to attend and stated that the people were selected at random. The 3 people selected were women. There were concerns that no men were selected to attend the convention.

(a) Calculate the probability that randomly selecting 3 people from a group of 6 men and 3 women will result in selecting 3 women.

probability of selecting 1 woman = $3/9 = 1/3$

2 women = $2/8$

3 women = $1/7$

$(1/3)(2/8)(1/7) = 1/84 = .0119$

(b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

There is reason to doubt the claim because there is a 1% chance that 3 women would be selected and this is highly unlikely.

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2 C2

(c) An alternative to calculating the exact probability is to conduct a simulation to estimate the probability. A proposed simulation process is described below.

> Each trial in the simulation consists of rolling three fair, six-sided dice, one die for each of the convention attendees. For each die, rolling a 1, 2, 3, or 4 represents selecting a man; rolling a 5 or 6 represents selecting a woman. After 1,000 trials, the number of times the dice indicate selecting 3 women is recorded.

Does the proposed process correctly simulate the random selection of 3 women from a group of 9 people consisting of 6 men and 3 women? Explain why or why not.

Yes. In order to correctly perform a simulation, the probabilities of obtaining a random number should be the same as attaining the corresponding action in real life. In this case, the probability of rolling a 5 or 6 is the same probability of selecting a woman at random. The probability is 1/3. The probability of rolling a 1, 2, 3, or 4 is the same as selecting a man; 2/3.

Selecting a woman — 3/9 = 1/3
rolling a 5 or 6 — 2/6 = 1/3

Selecting a woman = rolling a 5 or 6

Selecting a man — 6/9
rolling a 1, 2, 3, or 4 — 4/6 = 2/3

Selecting a man = rolling a 1, 2, 3, 4

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Question 2

Overview

The primary goals of this question were to assess a student's ability to (1) calculate a probability; (2) assess whether a claim about randomness is questionable in light of a calculated probability; and (3) judge whether a description of a simulation method achieves a correct simulation of a random process.

Sample: 2A
Score: 4

In part (a) the student gives the correct probability of 0.012 and shows how it was computed. Part (a) was scored as essentially correct. In part (b) the student correctly states that "there is reason to doubt the manager's claim" and supports this assertion by saying that "There is a small chance that these results would occur if the selection was random," which is appropriate for such a small probability of 0.012. The communication is very strong, using the conditional tense ("would occur"), and conditioning the probability upon the claimed random selection ("if the selection was random"). Part (b) was scored as essentially correct. In part (c) the student correctly states that "[t]he proposed process does not simulate the selection correctly." The student then correctly addresses the independence of the outcomes of the three dice by saying that "the trials are independent of each other," as well as by indicating that the proposed process "assumes the probability that a woman will be selected is 1/3." The response would have been stronger had the student indicated the probability was the same for all three selections, but the overall communication is clear enough that the phrase is implied. Finally, the student addresses the dependence of the genders of the selected convention attendees succinctly but clearly by concluding, "both of which are not true." The response in part (c) was scored as essentially correct. Because all three parts were scored as essentially correct, the response earned a score of 4.

Sample: 2B
Score: 3

In part (a) the computation shows a recognition that after each woman is selected, the number of remaining women decreases. This is indicated by the numerators of the three component fractions, which decrease from 3 to 2 to 1. However, the student fails to recognize that the total number of people remaining also should decrease from 9 to 8 to 7. Part (a) was scored as partially correct. In part (b) the student correctly states that "there is reason to doubt the manager's claim," and correctly supports this statement by saying that "it is *highly unlikely* that out of a pick of 3, all 3 women were chosen" (italics added). The response would have been stronger had the student used the conditional tense of *would have been chosen* rather than the past tense *were chosen*, since the probability requested in part (a) is not that of a past event, but of a conditional event—the selection of 3 women supposing that the selection is done at random. But that weakness in communication is sufficiently minor that it does not alter the score. Part (b) was scored as essentially correct. In part (c) the student correctly states that "the proposed simulation does not simulate the random selection." The student correctly addresses the independence of the dice outcomes by saying that "*each time* you roll a die you have a 2/6 or 1/3 chance of rolling a 5 or a 6" (italics added), which implies independence. The student contrasts this with the dependence of the genders of the selected convention attendees by stating that "in the actual random selection of men & women the probability of choosing a woman goes down after one has already been selected", which implies dependence. The overall communication in part (c) is quite strong—for example, through its use of the contrasting word "[h]owever." Part (c) was scored as essentially correct. Because two parts were scored as essentially correct and one part was scored as partially correct, the response earned a score of 3.

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Question 2 (continued)

Sample: 2C
Score: 2

In part (a) the student gives the correct probability of 0.0119 and shows how it was computed. The response includes a communication error, in that it states "2 women = $\frac{2}{8}$" and "3 women = $\frac{1}{7}$" rather than the more accurate $P(\text{2nd woman}) = \frac{2}{8}$ and $P(\text{3rd woman}) = \frac{1}{7}$. However, the error is minor and does not affect the score in part (a). Part (a) was scored as essentially correct. In part (b) the response correctly states that "[t]here is reason to doubt the claim" and supports this statement by saying that "this is highly unlikely," which is appropriate for the small probability of 1 percent. The response is strengthened by the use of the conditional tense ("would be selected"). The response would have been stronger if the student had indicated that the selection had actually been random, but the communication is strong enough that this is implicit. Part (b) was scored as essentially correct. In part (c) the student incorrectly states that the proposed process successfully simulates the selection of 3 women from a group of 9 people consisting of 6 men and 3 women. Although the student recognizes that the chance of selecting *one* woman and the chance of rolling a 5 or a 6 on *one* die are equal $\left(\frac{1}{3}\right)$, the student fails to recognize that there is dependence between the genders of the three selected convention attendees and does not contrast that with the independence of the dice outcomes. Part (c) was scored as incorrect. Because two parts were scored as essentially correct and one part was scored as incorrect, the response earned a score of 2.

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